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Sphaleron–bisphaleron bifurcations in a custodial-symmetric two-doublets model

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Abstract

The standard electroweak model is extended by means of a second Brout–Englert–Higgs doublet. The symmetry breaking potential is chosen in such a way that (i) the Lagrangian possesses a custodial symmetry, (ii) a static, spherically symmetric ansatz of the bosonic fields consistently reduces the Euler–Lagrange equations to a set of differential equations. The potential involves, in particular, products of fields of the two doublets, with a coupling constant λ_3. Static, finite energy solutions of the classical equations are constructed. The regular, non-trivial solutions with the lowest classical energy can be of two types: sphalerons or bisphalerons, according to the coupling constants. Special emphasis is put on the bifurcation between these two types of solutions which is analyzed in the function of the different constants of the model, namely of λ_3.

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1. Introduction

It has been known for a long time [1] that baryon and lepton numbers are not strictly conserved in the standard model of electroweak interactions (see [2, 3] for reviews). Baryon number violating processes [4] involve the crossing through an energy barrier separating topologically inequivalent vacua of the underlying gauge theory. Remarkably, this energy barrier is high but finite. It corresponds to a static, regular solution of the classical equations of motion: the sphaleron [5]. The sphaleron was first constructed in the case $\theta_W = 0$ (θ_W denotes the Weinberg angle) where a consistent spherically symmetric ansatz [6] transforms the Euler–Lagrange equations of the theory into differential equations. The Klinkhamer–Manton (KM) sphaleron is, however, not the minimal energy barrier when the mass of the Brout–Englert–Higgs boson (BEH boson) exceeds some critical values. Indeed, for $M_H \gg M_W$ [7–9] another branch of solutions exists which bifurcates from the sphaleron branch for $M_H \approx 12 M_W$. The new solutions have a lower energy than the sphaleron and since they appeared as pairs connected to each other by the parity operator, they were called *bishpalerons*. Nowadays the possibility that bisphalerons constitute the energy barrier allowing for baryon number violating process is ruled out in the minimal (one doublet) electroweak model by the perturbative upper limit of the BEH field and the sphaleron–bisphaleron bifurcation remains a curiosity of the classical equations.

However, several extensions of the minimal Weinberg–Salam model are currently under investigation as alternative candidates for the description of electroweak interactions (see e.g. [10, 11]). Among these various extensions, those incorporating more than one multiplet of BEH bosons play a central role. For instance, the minimal supersymmetric electroweak model, considered for many theoretical reasons, involves two BEH doublets. These extended models lead generally to involved classical equations where the generalizations of sphalerons and bisphalerons can be looked for, as well as eventual other types of solutions of soliton type. In particular, it is challenging to study the domain of parameters for which bisphalerons exist and to see if this domain intersects with the domain of physically acceptable parameters. This question was addressed before in [12, 13]. The potential used in these papers does not involve a direct coupling between the doublets. Here we will extend the potential chosen in [12, 13] by a supplementary interaction between the two BEH doublets. The influence of the new term on the sphaleron–bisphaleron bifuraction will be studied in details.

To be complete let us mention that the classical equations of the two-doublets-extended standard model were also investigated in [14, 15] with even more general potentials but, to our knowledge, these authors did not put the emphasis on the bifurcation between the two types of lowest energy solutions.

In section 2, we present the model, the notations and the physical parameters. The spherically symmetric ansatz, the equations and boundary conditions are given in section 3; the numerical solutions are then discussed in section 4.

2. The model

The model that we consider in this paper is an SU(2) Yang–Mills theory coupled to two doublets of scalar fields. The Lagrangian density is given by

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu,a} + (D_\mu\Phi_{(1)})^\dagger(D^\mu\Phi_{(1)}) + (D_\mu\Phi_{(2)})^\dagger(D^\mu\Phi_{(2)}) - V(\Phi_{(1)},\Phi_{(2)}), \tag{1}$$

where $\Phi_{(1)}, \Phi_{(2)}$ denote the two BEH doublets and the standard definitions are used for the covariant derivative and gauge-field strengths:

$$F^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu + g\epsilon^{abc}V^b_\mu V^c_\nu \tag{2}$$

$$D_\mu\Phi = \left(\partial_\mu - \frac{\mathrm{i}}{2}g\tau^a V^a_\mu\right)\Phi \tag{3}$$

where τ_a denotes the Pauli matrices and g the gauge coupling. With respect to the electroweak model, the limit $\theta_W = 0$, i.e. $g' = 0$, is used throughout the paper.

The most general gauge invariant potential constructed with two BEH doublets is presented namely in [11], it depends on nine constants. Here we consider the family of potentials of the form

$$V(\Phi_{(1)},\Phi_{(2)}) = \lambda_1\left(\Phi^\dagger_{(1)}\Phi_{(1)} - \frac{v_1^2}{2}\right)^2 + \lambda_2\left(\Phi^\dagger_{(2)}\Phi_{(2)} - \frac{v_2^2}{2}\right)^2 + \lambda_3\left(\Phi^\dagger_{(1)}\Phi_{(1)} - \frac{v_1^2}{2}\right)\left(\Phi^\dagger_{(2)}\Phi_{(2)} - \frac{v_2^2}{2}\right) \tag{4}$$

depending on five parameters. The term directly coupling the two doublets is parametrized by the constant λ_3. One of the main properties of the potential (4) resides in the fact that it imposes a symmetry breaking mechanism to each of the BEH doublets. The case $\lambda_3 = 0$ is studied at length in [12, 13].

The Lagrangian (1) is invariant under SU(2) gauge transformations but it further possesses a larger global symmetry under SU(2) × SU(2) × SU(2). In fact, the part of the Lagrangian (1) involving the scalar fields can be written in terms of 2 × 2 matrices defined by

$$M_a \equiv \begin{pmatrix} \phi_0^* & \phi_+ \\ -\phi_+^* & \phi_0 \end{pmatrix}_a \qquad \text{with} \qquad \Phi_{(a)} \equiv \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix}_a, \qquad a = 1, 2. \tag{5}$$

When written in terms of the matrices M_1 and M_2, the Lagrangian (1) becomes manifestly invariant under the transformation

$$V'_\mu = AV_\mu A^\dagger, \qquad M'_1 = AM_1B, \qquad M'_2 = AM_2C \tag{6}$$

with $A, B, C \in \mathrm{SU}(2)$; this is the custodial symmetry. The double symmetry breaking mechanism imposed by the potential (4) leads to a mass M_W for two of the three gauge vector bosons and, namely, to two neutral BEH particles with masses M_h, M_H. In terms of the parameters of the Lagrangian, these masses are given by [11, 16]

$$M_W = \frac{g}{2}\sqrt{v_1^2 + v_2^2}, \qquad M_{H,h}^2 = \frac{1}{2}\left[A_1 + A_2 \pm \sqrt{(A_1 - A_2)^2 + 4B^2}\right] \tag{7}$$

with

$$A_1 = 2v_1^2(\lambda_1 + \lambda_3), \qquad A_2 = 2v_2^2(\lambda_2 + \lambda_3), \qquad B = 2\lambda_3 v_1 v_2. \tag{8}$$

For later convenience we also define

$$\tan\beta = \frac{v_2}{v_1}, \qquad \rho_{H,h} = \frac{M_{H,h}}{M_W}, \qquad \epsilon_p = 4\frac{\lambda_p}{g^2}, \qquad p = 1, 2, 3. \tag{9}$$

Note that the quantities $\rho_{1,2}$ used in [13] are related to the mass ratio $\rho_{H,h}$ by $\rho_H = \max\{\rho_1, \rho_2\}, \rho_h = \min\{\rho_1, \rho_2\}$. For physical reasons, we consider only $v_1 \geqslant 0, v_2 \geqslant 0$ so that $0 \leqslant \beta \leqslant \pi/2$. Interestingly, the parameter ϵ_3 can be negative but cannot take arbitrary values. The following relations are useful to determine the physical region:

$$\begin{aligned} \epsilon_1 \cos^2\beta + \epsilon_2 \sin^2\beta &= \tfrac{1}{2}\left(\rho_H^2 + \rho_h^2\right) - \epsilon_3, \\ \epsilon_1 \cos^2\beta - \epsilon_2 \sin^2\beta &= \tfrac{1}{2}\sqrt{\left(\rho_H^2 - \rho_h^2\right)^2 - 4\epsilon_3^2 \sin^2(2\beta)} - \epsilon_3 \cos(2\beta). \end{aligned} \tag{10}$$

The physical domain is then determined by the conditions

$$\frac{\rho_h^2 - \rho_H^2}{2\sin(2\beta)} \leqslant \epsilon_3 \leqslant \frac{\rho_H^2 - \rho_h^2}{2\sin(2\beta)}, \qquad 0 \leqslant \rho_h^2 \leqslant \rho_H^2 \tag{11}$$

The physical domains are presented in figure 1 for the case $\rho_h = 3$ for $\beta = \pi/4$ and $\beta = 3\pi/8$.

3. Spherical symmetry

In order to construct classical solutions of the Lagrangian (1), we perform a spherically symmetric ansatz for the fields. With the notations of [17], it reads as

$$\begin{aligned} V_0^a &= 0 \\ V_i^a &= \frac{1 - f_A(r)}{gr}\epsilon_{aij}\hat{r}_j + \frac{f_B(r)}{gr}(\delta_{ia} - \hat{r}_i\hat{r}_a) + \frac{f_C(r)}{gr}\hat{r}_i\hat{r}_a \end{aligned}$$

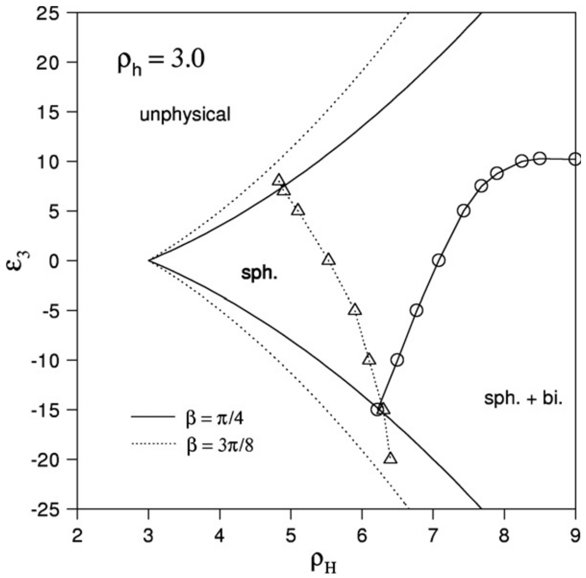

Figure 1. The physical domain corresponding to the case $\rho_h = 3.0$, $\beta = \pi/4$ (resp. $\beta = 3\pi/8$) are represented by the solid (resp. dashed) line in the ρ_H, ϵ_3 plane. The line with bullets (resp. triangle) represents the sphaleron–bisphaleron bifurcation.

$$\phi_{(1)} = \frac{v_1}{\sqrt{2}}[H(r) + \mathrm{i}(\hat{r}.\vec{\sigma})K(r)]\begin{pmatrix}0\\1\end{pmatrix}$$
$$\phi_{(2)} = \frac{v_1}{\sqrt{2}}[\tilde{H}(r) + \mathrm{i}(\hat{r}.\vec{\sigma})\tilde{K}(r)]\begin{pmatrix}0\\1\end{pmatrix}, \tag{12}$$

where $f_A, f_B, f_C, H, K, \tilde{H}, \tilde{K}$ are real functions of the radial coordinate $r = \sqrt{x^2 + y^2 + z^2}$. It can be shown that the above ansatz transforms the Euler–Lagrange equations into a set of coupled differential equations. The custodial symmetry has been used to set the two doublets parallel to each other asymptotically. The condition $V_0 = 0$ results from a gauge fixing. In fact, the spherically symmetric ansatz leaves a residual gauge symmetry which can be exploited to eliminate one of the seven radial functions [7, 8, 17]. Here we will adopt the radial gauge $x_j V_j^a = 0$ which implies $f_C = 0$.

The classical energy E of a static, spherically-symmetric configuration can be computed by substituting the fields (12) into the energy functional. After some algebraic manipulations, the quantity E can be written in the form:

$$E = \frac{M_W}{\alpha_W}\tilde{E}, \qquad \tilde{E} = \int_0^\infty \mathcal{E}\,\mathrm{d}x. \tag{13}$$

Here, $\alpha_W \equiv \frac{g^2}{4\pi}$ and the effective one-dimensional energy density $\mathcal{E}$ reads

$$\begin{aligned}\mathcal{E} = {} & (f_A')^2 + (f_B')^2 + \frac{1}{2x^2}\left(f_A^2 + f_B^2 - 1\right)^2 \\ & + \cos^2\beta[(H(f_A-1) + Kf_B)^2 + (K(f_A+1) - Hf_B)^2 + 2x^2((H')^2 + (K')^2) \\ & + (\tilde{H}(f_A-1) + \tilde{K}f_B)^2 + (\tilde{K}(f_A+1) - \tilde{H}f_B)^2 + 2x^2((\tilde{H}')^2 + (\tilde{K}')^2)] \\ & + \cos^4\beta[\epsilon_1 x^2(H^2 + K^2 - 1)^2 + \epsilon_2 x^2(\tilde{H}^2 + \tilde{K}^2 - \tan^2\beta)^2 \\ & + \epsilon_3 x^2(H^2 + K^2 - 1)(\tilde{H}^2 + \tilde{K}^2 - \tan^2\beta)].\end{aligned} \tag{14}$$

The dimensionless variable $x = M_W r$ is used and the prime denotes the derivative with respect to x. The equations to solve can then be obtained by varying the functional (14) with respect to the six radial functions. Remark that in the case $v_2 = 0, \lambda_2 = \lambda_3 = 0$ the equations for $\tilde{H}, \tilde{K}$ decouple and these functions can be set consistently to zero; the remaining equations then correspond to the case of one doublet.

The conditions of regularity of the solutions at the origin imposes in particular $f_A^2 + f_B^2 = 1$ at $x = 0$, the custodial symmetry (6) can then be exploited to fix the following values of the radial fields at the origin:

$$\begin{aligned} &f_A(0) = 1, \qquad f_B(0) = 0, \qquad K(0) = 0, \\ &\tilde{K}(0) = 0, \qquad H'(0) = 0, \qquad \tilde{H}'(0) = 0. \end{aligned} \tag{15}$$

On the other hand, the condition of finiteness of the classical energy imposes the following asymptotic forms:

$$\begin{aligned} (f_A, f_B)_{x=\infty} &= (\cos 2\pi q, \sin 2\pi q) \\ (H, K)_{x=\infty} &= (\cos \pi(q-k), \sin \pi(q-k)) \\ (\tilde{H}, \tilde{K})_{x=\infty} &= \tan\beta(\cos \pi q, \sin \pi q) \end{aligned} \tag{16}$$

for some real number q and for k equal to zero or one. For later use we define $q \equiv 1/2 + \delta$.

4. Discussion of the solutions

In order to make the following discussion self-contained, we first summarize the main features of the solutions available in the case of the one doublet-standard-model (1DSM), i.e. in the case $v_2 = \lambda_2 = \lambda_3 = 0$, leading to $\tilde{H} = \tilde{K} = 0$.

4.1. 1DSM

There exists at least one solution for all values of ρ_1: the Klinkhamer–Manton (KM) sphaleron [6]. Note that $\rho_H \equiv \rho_1$ in this case. For this solution one can further set $f_B = H = 0$ by an appropriate choice of the custodial symmetry; the classical energy increases monotonically as a function of ρ_1:

$$\tilde{E}_s(\rho_1 = 0) \approx 3.04, \qquad \tilde{E}_s(\rho_1 = \infty) \approx 5.41. \tag{17}$$

The KM sphaleron is always unstable but the number of its directions of instability increases when ρ increases [8, 18]. At $\rho \approx 12.04$ a couple of new solutions, the bisphalerons, bifurcate from the sphaleron. The two bisphalerons (which transform into each other by parity) have the same energy and their energy is lower than that of the KM sphaleron

$$\tilde{E}_{bs}(\rho_1 = 12.04) = \tilde{E}_s(\rho_1 = 12.04) \approx 4.86, \qquad \tilde{E}_{bs}(\rho_1 = \infty) \approx 5.07. \tag{18}$$

The parameter q defined in (16) is equal to 1/2 for sphalerons. For bisphalerons, the parameter q deviates slightly from $q = 1/2$. Defining $\delta \equiv q - 1/2$ it is known [7, 8] that δ varies from zero (at the bifurcation point) to $\delta = \pm 0.06$ (at $\rho_1 = \infty$). The two bisphalerons are distinguished by the sign of δ.

4.2. 2DSM, case $\epsilon_3 = 0$

Solutions of the Lagrangian (1) with $\lambda_3 = 0$ were first constructed in [12] and reconsidered in [13] where the emphasis was put on the sphaleron–bisphaleron bifurcation. As in 1DSM, sphalerons have $f_B = H = \tilde{H} = 0$ and seem to exist for all values of the parameters of the

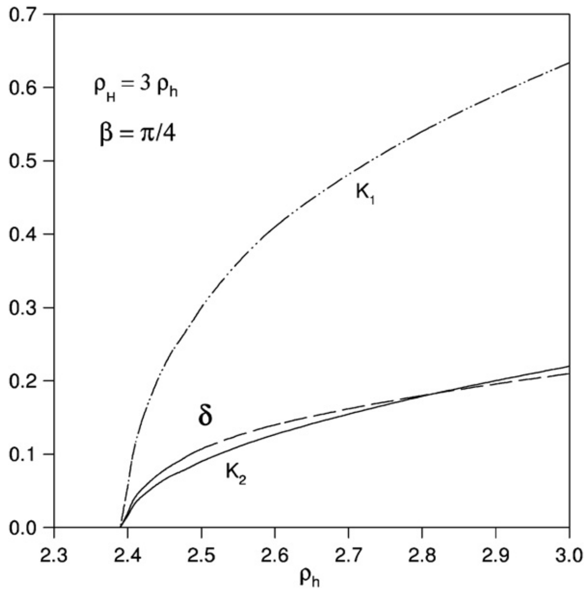

Figure 2. The evolution of the parameters δ, $K_1 \equiv K(0)$, $K_2 \equiv \tilde{K}(0)$ is shown as function of ρ_h for $\rho_H = 3\rho_h$.

potential. The angle parameter q defined in (15) corresponds to $q = 1/2$, irrespectively of the coupling constants of the potential.

By contrast, the six radial functions corresponding to the bisphaleron are non-trivial and fulfil the boundary conditions (16). The parameter q depends on the various coupling constants but remains close to $1/2$ for all choices (e.g. $\delta = 0.036$ for $\rho_1 = 14$, $\rho_2 = 1$, $\beta = 0.2$).

The results of [13] show the existence of a smooth surface in the ρ_1, ρ_2, β-parameter space inside of which only sphaleron solutions exist while sphalerons and bisphalerons coexist outside, the bifurcation taking place *on* the surface. The critical surface can be determined only numerically by studying a few parameters characterizing the bisphaleron solutions, namely the values δ, $K(0)$, $\tilde{K}(0)$, as functions of $\beta, \epsilon_1, \epsilon_2$. Varying one of these parameters and fixing the other two, a critical point is determined when δ, $K(0)$, $\tilde{K}(0)$ approach zero. This is illustrated in figure 2 for $\beta = \pi/4$ and $\rho_H = 3\rho_h$. The critical point then corresponds to $\rho_H \approx 2.397$.

It is worth noticing that bisphaleron solutions with $\delta = 0$ but $K(x) \neq 0$, $\tilde{K}(x) \neq 0$ also occur outside the critical surface. This is illustrated in figure 3 where we set $\rho_H^2 + \rho_h^2 = 9$ and vary $\Theta \equiv \arctan(\rho_H/\rho_h)$. Clearly, δ goes to zero in the limit $\Theta \to \pi/4$. Figures 2 and 3 are complementary to those presented in [12, 13].

One important feature of the bisphaleron solutions in the 2DSM is that the angle $\tilde{\phi} = \arctan(\tilde{K}/\tilde{H})$ increases monotonically from 0 (for $x = 0$) to πq (for $x = \infty$) while $\phi = \arctan(K/H)$ decreases from 0 to $\pi(q-1)$; that is to say that these solutions of lowest energy have $k = 1$ in (16). In fact, non-trivial solutions obeying (16) with $k = 0$ were constructed in [12] but since they have a higher energy, they are likely to be less interesting as far as the energy barrier is concerned.

4.3. 2DSM, case $\epsilon_3 \neq 0$

The construction of solutions in the domain $\rho_h, \rho_H, \beta, \epsilon_3$ and the study of the critical hypersurface in this four-parameter space is a vast task. For definiteness, we have limited our investigations to two values of β, namely $\beta = \pi/4$ and $\beta = 3\pi/8$.

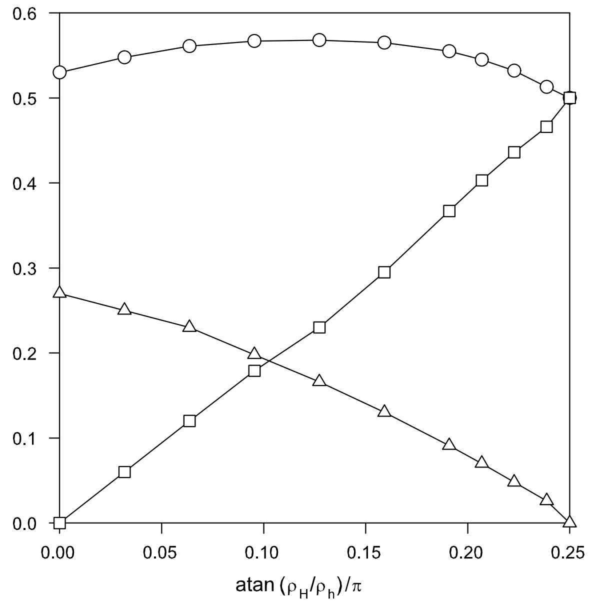

Figure 3. The evolution of the parameters δ, $K_1 \equiv K(0)$, $K_2 \equiv \tilde{K}(0)$ is shown as function of ρ_H/ρ_h for $\rho_H^2 + \rho_h^2 = 9$.

We first discuss the results for $\beta = \pi/4$, i.e. $v_1 = v_2$. Here are a few 'points' on the bifurcation line corresponding to $\beta = \pi/4$ and $\epsilon_3 = 0$:

$$(\rho_h, \rho_H) : (3.0, 7.08), (5.0, 6.08), (5.585, 5.585), (7.6, 10). \quad (19)$$

Our main concern is to determine how this domain of the ρ_h, ρ_H plane evolves with ϵ_3. In the following, we further restrict to the case $\rho_h = 3.0$. The result is illustrated by figure 1, where the physical domain is delimited by the solid lines. The sphaleron–bisphaleron bifurcation line is represented by the solid line with bullets and the domains where sphalerons only exist is indicated, as well as the domain where sphalerons and bisphalerons coexist. The critical line separating these two regions clearly exhibits two different behaviors on the domain of parameters considered: for $6.2 \leqslant \rho_H \leqslant 8$, the critical line is roughly a function increasing linearly with the coupling constant ϵ_3. As a consequence, the critical line intersects the lower line delimiting the physical domain, for instance at $\rho_H \approx 6.22$, $\epsilon_3 \approx -15.0$. Clearly, for the negative values of ϵ_3, the minimal mass of M_H (with all other parameters fixed) for which bisphalerons exist is lowered by the presence of a direct interaction between the BEH fields. For $8.0 < \rho_H < 10.0$ the critical line develops a plateau at $\epsilon_3 \approx 10.0$ and $\rho_{H,\mathrm{cr}}$ depends only weakly of ϵ_3. For some unknown reason, the numerical analysis becomes very difficult when the critical line approaches the limit of the physical domain.

The behavior of the critical line turns out to be completely different for $\beta = 3\pi/8$, in this case, the limit of the physical domain is indicated by the dotted lines and the critical line by the dotted line with the triangles. In contrast to the case $\beta = \pi/4$ we see here that the critical value $\rho_{H,\mathrm{cr}}$ decreases roughly linearly with ϵ_3. For $\epsilon_3 > 0$ the barrier turns out to be a bisphaleron for lower values of ρ_H than in the $\epsilon_3 = 0$ case. Here, the critical line seems to cross the two lines determining the physical domain where it naturally terminates.

We further studied the critical phenomenon for $3\pi/8 < \beta < \pi/4$ and observed a smooth evolution of the critical lines displayed in figure 1.

5. Conclusion

The Lagrangian considered in this paper leads to a tricky system of six differential equations with boundary conditions and depending effectively on four parameters. Many types of non-trivial solutions can be constructed numerically [12] but, at the moment, those with lowest energy are identified as the sphaleron or the bisphaleron, depending on the different coupling constants. The determination of the critical hypersurface of bifurcation in the space of parameters constitutes a huge task which can be studied only numerically. The problem is for a large part academic; however at the moment, the theoretical limits on the BEH-boson masses obtained in the two-doublets extension of the electroweak model, do not yet exclude that the energy barrier between topologically different vacua could be determined by a bisphaleron. A few years ago, it was already pointed out that the minimal mass of the neutral BEH fields for the barrier to be of the bisphaleron type is considerably lower in the two-doublets model (without direct interaction of the doublets in the potential) than in the minimal, one-doublet model. The calculations reported here suggest that, if a custodially-invariant coupling term is supplemented to the potential, the critical mass $\rho_{H,\mathrm{cr}}$ varies roughly linearly with the new coupling constant ϵ_3. The supplementary coupling constant can of course not be arbitrarily large, but on the physical domain, the critical value $\rho_{H,\mathrm{cr}}$ again decreases while all other masses are kept fixed. Of course, many other terms can be added to the potential [11]; namely the terms leading to charged BEH fields. It could be that bisphaleron solutions exist for still lower masses of the different scalar particles emerging from the BEH mechanism.

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